Mathematics, for the student who uses braille, commences in the early grades with an emphasis on the use of a variety of concrete, functional manipulatives. Gradually the Braille Writer (Perkins), to record data, and the Abacus and talking calculators to compute problems, are introduced and become major tools at the elementary level. The introduction of the Nemeth code starts in kindergarten and its mastery is a focus right through to graduation. At the secondary level students who use braille are building on the knowledge of the Nemeth Code, accessing and recording information, working on independence and problem solving. Additional skills are developed with the introduction and use of the talking scientific calculator (Braille 'n Speak 640, Braille Lite or a talking computer) and the Pocket Slate and Stylus. At all levels of instruction from Kindergarten to graduation the student who uses braille takes longer than his or her sighted peers to read and record information.

Students from K 12 should actively be working towards using and mastering one or a combination of the Abacus, Braille Writer (Perkins), Braille Lite, Braille 'n Speak 640 and talking calculator. It is important to note that the Braille Writer (Perkins) is especially useful throughout all the school years, as it is the only tool that allows a student to show all of the steps in their work.

Pre-teaching new concepts, skills and Nemeth symbols, by a Vision Resource Teacher, prior to regular classroom introduction, allows the student who uses braille the time to learn and practice on a one to one basis. Thus, the student can focus on the lesson in the classroom instead of classroom instruction plus learning how to manipulate unique materials and technology. Extra time must be found for the student to master these additional skills.
Students who use braille should have equal opportunity and access to the same print materials as their sighted peers. Regular meetings between the Vision and Classroom Teacher are required to discuss and plan the instructional program, priorities, and time required for the production of alternate format materials.

Many math concepts can be reinforced by twin vision books (both braille and print) and adapted board games made for classroom activities. These provide the student with additional incidental practice with peers, in a functional setting, which is needed to reinforce concepts. The abacus can be used to keep score for any number of games in the classroom.

Whenever possible concrete, functional materials should be used to teach math concepts. For example, if the student is required to count the number of frogs for one to one correspondence, the student who uses braille may count the number of Nemeth symbols. This allows for some incidental Nemeth code learning as well as one to one correspondence tasks. These are meaningful activities for the student who uses braille. Emphasis on concrete information can be carried over into the area of orientation and mobility (O&M). For example walking around the outside of a playground can reinforce the concept of perimeter and $90^\circ$ angles.

The Vision Resource Teacher in your school district is the most knowledgeable in adapting and providing materials for the student who uses braille. Together as a team, the classroom teacher provides the math knowledge, and the vision teacher provides the adapted materials and direct instruction for specialized skills such as the Nemeth Code.

Each of the devices discussed in this section has advantages and disadvantages. Often technology ideal for one task will not be ideal for another. It is imperative that the student be introduced to a variety of technology so that he or she may learn what works best.
Visually Impaired Resource Guide
Mathematics Kindergarten and Grade One

At this level children are given objects such as blocks and counting sticks to work and play with. Children learn how to make simple measurements and how to use pictures, numbers, shapes and sizes and simple mathematical symbols such as plus and minus.

It is very important at this level to give the student a variety of manipulatives to teach and reinforce one to one correspondence, to use concrete objects for simple measurements, to learn to describe the attributes of an object and to learn to discriminate differences in size and weight. Braille instruction should commence in writing and reading the number sign, Nemeth* numbers, plus, minus and equal signs. Additional time will need to be spent in learning how to manipulate the brailler and the process of putting in, aligning and removing braille paper. If calculators are available, the visually impaired student should have access to experience and use a calculator with speech output. These experiences with electronic technology must not replace necessary experiences using manipulatives or the brailler. The introduction of skills should parallel classroom instruction as closely as possible.

Blind students must have physical boundaries when dealing with manipulatives (e.g. trays, sorters, cups, egg cartons) to prevent items from getting lost or being placed in the wrong group. This system also encourages the use of a systematic search technique.* (refer to the Vision Resource Teacher in your area for further information) One working group of manipulatives should have boundaries within the larger working tray. For example, when beginning the concept of an equation each portion of the task should have well defined containers/boundaries.

It is important to adapt tasks so they are meaningful to the visually impaired child. For example, if the task is to count the rings on a picture of a finger and write the corresponding number, the blind child could count the number of Nemeth number signs or embossed lines or shapes or other tactile attributes on a braille page and write the corresponding number. The task is to count, therefore count something meaningful to the blind child. The added bonus is often in facilitating incidental learning such as refining tactile discrimination skills.
Learning About Numbers

- learn about whole numbers and fractions
- know how to estimate the number of objects and add and subtract numbers up to 10
- know how to count up to 100 and begin to understand place value up to 99
- add and subtract to solve simple problems

The student should develop a systematic search technique (refer to the Vision Resource Teacher in your area for further information) location terminology and related concepts. (e.g. top of page, bottom of page, left, right) Use weight, sound, and size as estimators of the number of objects in a group. The abacus and a combination braille/tactile/large print number line, can be used to teach and reinforce one to one correspondence, place value and simple addition and subtraction problems.

Data Analysis

- Collect information and show how this information fits into different categories (displaying)

Continue to use a variety of meaningful manipulatives to represent visual information and to achieve these curriculum goals. This is the very beginning of charting and graphing. Students should learn how to read and produce information on a simple chart and be exposed to some basic formatting techniques. Tactile graph paper is available commercially.

Geometry

- learn about objects and shapes such as circles, triangles and squares
- learn about geometric patterns using shapes and objects

Continue to use a variety of meaningful manipulatives to achieve these curriculum goals. Learn the basic vocabulary and attributes of shapes. (e.g. circles, triangles and squares) Relate these shapes and attributes to real life objects whenever possible.

Learning About Measurements

- compare, estimate and record the length of things using centimeters (standard units) as well as objects such as hands, feet and pencils (non-standard units)
- begin to know about time, temperature and money

The use of real money for the blind child helps with daily living skills and the Vision Teacher may be consulted in the instruction of identifying coins tactually and folding techniques to identify paper bills. Tools that can be used at this level include the talking clock, Braille Lite * (refer to the Vision Resource Teacher in your area for further information) clock, raised clock, talking thermometer, as well as commercially prepared worksheets.

Algebra

- Algebra is not part of the mathematics program in K to 3
Children are now-learning how adding, subtraction, multiplying and dividing can be used in daily activities and for solving problems. They learn to collect information and to display it using pictures and symbols. Students learn the basic facts of addition, subtraction and multiplication, as well as ways of estimating.

The brailler and abacus are used regularly in conjunction with manipulatives and a tactile/braille/large print number fine. These should become the daily tools for solving problems and learning how to record and read Nemeth information. Students learn to collect and display information using representational tactile symbols.

Learning and practicing how to manipulate braille paper and the brailler, for calculation purposes, needs to be mastered. Considerable time is necessary at this level to teach the student how to work with the Nemeth format and how to represent the operations and calculations on a braille page.

Please refer to the Math Overview for additional information.

**Learning About Numbers**

Extend counting, estimating, adding and subtracting the three numbers and place value to four numbers begin to learn about decimals and basic multiplication and division facts start to learn about fractions and how they can be written differently as equivalent fraction and decimals (e.g., $1/2 = 4/8 = 0.5$) continue to solve addition and subtraction problems with whole numbers.

Estimation is a very important skill when using the calculator and should be stressed from this level on upward. Place value provides an opportunity to teach the abacus to the entire class and reinforces this concept for all students. An added benefit is that at least one student will likely enjoy and understand the abacus to the level of being a resource for the visually impaired student. The abacus is an excellent tool for keeping score when playing games.
When using the abacus, comma and decimal point inserters can be used to help with calculations. A braille set of multiplication and division facts are useful tools for the home and classroom. A screen board and tactile card games are excellent ways to reinforce math facts and involve peer groups. These activities can be used for indoor days and help facilitate integration. Drilling math facts can become an independent activity with the use of a specialized tape recorder (Foto-Fonics) for tactile and auditory recording.

Fractions are introduced at this level and time will need to be spent in reading and recording fractions using the symbols and rules of the Nemeth code. Oakmount materials, provided at no cost for visually impaired students, are excellent tactile tools for the teaching of fractional concepts.

**Data Analysis**

- extend collecting and displaying data to include tallies (totals by category)
- use tables and pictographs to solve problems.

Continue to use manipulatives but gradually shift to braille representation of data. Commercially produced and teacher prepared materials can be useful in dealing with tables to solve problems. Whenever feasible use meaningful concrete materials.

**Geometry**

- identify, describe and classify more complex figures and objects
- learn the names of figures and shapes (e.g., circle, triangles, square)
- begin to learn about patterns a figure makes when it moves around

Students will learn how to describe the attributes of figures and shapes. (i.e. triangles have 3 pointed corners and the sides are all straight lines) A screen board or geoboard are useful tools when teaching and reinforcing geometry concepts. A brailler can be used to make Nemeth numbers and symbols creating pattern worksheets with prediction tasks at the end.
Learning About Measurements

- learn about area, capacity (volume), mass (weight) and temperature
- compare, estimate, measure and record, using common metric units and measuring tools
- begin to solve problems involving money, length, capacity (volume)

Buddy systems for experiments and teacher descriptions are good tools at this level of instruction. Tactile measuring devices are available from the Provincial Resource Centre for the Visually Impaired (PRCVI) to help with measurement. (refer to the Vision Resource Teacher in your area for further information.) The brailler continues to be the main tool of recording Nemeth information and instruction time will need to be set aside to learn the Nemeth rules for problems involving money, length, and capacity. It is helpful at this stage to continue to use real money and as many concrete environmental examples of measurement as possible.

Algebra

- Algebra is not part of the mathematics program in K to 3
At this level students use and build on their knowledge of math operations, data analysis, problem solving and measurements.

At this stage students who use braille build on their knowledge of the Nemeth Code* and formatting rules. Gaining proficiency on the abacus and the use of the Nemeth Code is an ongoing process that spans many years. New Nemeth symbols can be pre-taught or simultaneously taught as new concepts are introduced in the classroom. Functional manipulatives should continue to be used to teach and reinforce new skills.

**Learning About Numbers**

- count, estimate, add and subtract, using four-digit numbers
- extend whole number place value to five digits and estimation, addition and subtraction to four digits start to do some mental arithmetic
- do basic multiplication and division using whole numbers

Braille math formatting becomes very important when recording information and problem solving using the brailler as the recording tool. The unique aspects of recording math information and formatting on the brailler will require time to learn how to master and manipulate the brailler and braille page for spatial formats. (i.e. division)

To assist with the introduction of mental calculations, various methods can be taught to the student who uses braille to assist with this skill. Techniques such as "Finger Math" available from PRCVI* and Chisinhop (a method of using one's hands much like the abacus to make calculations) are only two of many ways to accomplish this goal.

**Data Analysis**

- gather information by conducting classroom poll or survey
- extend skills in reading and understanding displayed information (data) and use these skills to solve problems

Tactile graphing can be introduced at this stage. It is important to note that too much information on one graph, similar to map work, can be confusing to
the student just learning this new skill. Tactile aids are available and can supplement teacher constructed materials. Common resources such as tactile graph paper, geoboards, cuisinart rods and three dimensional models can be used to assist in the concept development and recording of information. Graphing is a two part process, the student must first learn how to record information and then how to read and interpret data from an already existing graph.

**Geometry**

- learn about lines and polygons and more about symmetry and pattern
- begin to learn about identical (congruent) shapes and objects and also about using number lines and grids as models for graphing

Tactile diagram kits, number lines and models are available to teach and reinforce geometric concepts. The area of orientation and mobility (refer to the Vision Resource Teacher in your area for further information) affords an excellent functional environment to augment many of these objectives.

**Learning About Measurements**

- start to measure volume
- start to make better estimations and solve problems involving time and temperature

Use a variety of functional and tactile tools for measuring. A talking thermometer, containers with tactile markings for Volume, a talking clock, the Braille Lite* refer to the Vision Resource Teacher in your area for further information) clock, the Braille 'n Speak* clock (refer to the Vision Resource Teacher in your area for further information) , the tactile clock, tactile rulers, meter sticks and commercially prepared worksheets are all devices that can be used at this level. Note that when sighted children look at various containers they will most often choose the tallest vessel to have the most volume (Piaget's theory on conservation) whereas the tactile learners will most often choose the fattest or widest container as holding the most. An excellent discussion comparing the theory of conservation between blind and sighted children can be found in Blindness and Early Childhood Development, 2nd Edition, Revised, by David H. Warren, published by the American Foundation for the Blind.
At this level students practice estimating and use this skill to check their answers. They try new ways of organizing and displaying data and begin to see how probability (chance) works. Students use some basics of geometry and learn how to use formulas to solve problems involving area and volume.

Building upon the student's knowledge of symbols in the Nemeth Code* emerges as students try new ways of organizing and displaying data. Instruction in the use of talking calculators can speed up calculations. The student must have mastered the concepts and accurately performed the operations of addition, subtraction, multiplication, and division in long hand before they can rely on the calculator. While mental calculation is encouraged the student should recognize his/her limitations in using this strategy with very large numbers. Braille labels can be adhered to the calculator keys, much the same as braille key caps for the typing keyboard. This can facilitate location of the correct keys and helps with speed and accuracy.

At this level students continue to practice estimating and use this skill to check their answers. Estimation will greatly assist the student who uses braille and time should be spent on mastering this skill.

**Learning About Numbers**

- estimate and calculate the answers to multiplication and division questions, using whole numbers and decimal fractions

- learn to divide, using two-digit divisors (e.g. 423025) and divide and multiply decimals by a one-digit whole number (e.g. 32.1606)

- learn about factors and multiples of whole numbers and improve mental arithmetic skills

Students should actively be using one or a combination of the abacus, brailler, Braille Lite*, Braille 'n Speak* and talking calculator to answer questions. The brailler is especially useful as it is the only tool that allows a student to show all of the steps in their work.

To assist with the mastery of mental calculations, techniques such as "Finger Math" (PRCVI *) and Chisinbop (a method of using one's hands much like the abacus to make calculations) can be useful:
Data Analysis

- do probability (chance) experiments
- learn about displaying and interpreting data to solve problems

For experiments, it may be helpful to pair the student with a sighted peer for visual observations. If possible, the peer should be a good math student with an understanding of the concepts and objectives. The Vision and Classroom Teacher need to meet regularly to discuss the instructional program and allow time for the production of braille materials. This is especially important when dealing with geometry and measurement units. Materials that are prepared in advance of the lessons allow pre-teaching, which is an excellent way of preparing the student for the concepts that will be taught in the classroom. In order to analyze data efficiently, the information must be displayed in an organized and legible format. The student may require additional instructional, time to master these skills.

Geometry

- earn about parallel and perpendicular lines
- earn more about patterns, using symmetry, identical figures and grids
- earn how to describe three-dimensional objects and points on a graph using ordered number pairs

If a student is having difficulty with geometric concepts, orientation and mobility* (refer to the Vision Resource Teacher in your area for further information) 1 instruction can provide a functional and physical way of experiencing and acquiring the concepts related to shapes. When working with graphs do not attempt to put too much information on any one graph. It is better to have three or four simple graphs than one with multiple themes.

Learning About Measurements

- learn about perimeter and area of shapes
- start to use formulas to find simple areas and volumes (e.g. area = l x w)

A variety of manipulatives, tactile diagrams and adapted math tools can be used to learn about perimeter and area of shapes. The talking calculator is a quick way to work with formulas associated with measurement.

Algebra

- write number sentences, using words and symbols
- use variables in expressions and formulas

Algebraic expressions and formulas require learning new Nemeth Code symbols and braille formatting. Extra instructional time may be needed for the student to learn, the Nemeth Code symbols and rules that are required to represent these concepts.
Visually Impaired Resource Guide
Mathematics Grade Six

At this level students write and talk about math and learn new mathematical terms. They are shown how to use algebra to solve problems and gain more knowledge of geometric relationships. They learn how to divide with decimals and to add and subtract fractions. They learn more about algebra and estimation and they check answers.

The Grade 6 curriculum places an emphasis on the use of calculators. The student who uses braille must develop calculator skills. Students can have access to a number of electronic calculators such as the stand atone talking calculator, the Braille 'n Speak,* Braille Lite * calculator and the built in computer calculator. Concurrent with mastery of the calculators the student will need to continue learning and building upon Nemeth symbols and algebraic expressions and formulas. Estimation continues to be an important skill in checking answers and the calculator becomes a functional tool in this area.

Many math concepts can be reinforced by twin vision books (both braille and print) and adapted board games that can be used in the educational environment. This gives the student additional incidental practice with peers in a functional setting that is needed to reinforce concepts. The abacus or any of the electronic calculators can be used to keep score for classroom games.

Learning About Numbers

• learn how to divide using decimals divisors
• learn how to add and subtract common fractions (sums to 1) and multiply fractions by a whole number
• begin to learn about order of operations (e.g., $3 + 7 \times 2 = 17$)
• solve problems using ideas developed at previous levels

Introduction of new concepts should parallel the regular curriculum.

Data Analysis

• learn to expand data collection through developing questionnaires and probability experiments (looking at expected patterns such as the number of times the same number will show up when you roll dice)
• learn how to draw conclusions from graphs
The Vision and Classroom Teacher need to meet regularly to discuss the instructional program and allow time for the production of braille materials. Materials that are prepared in advance of the lessons allow pre-teaching, which is an excellent way of preparing the student for the concepts that will be taught in the classroom.

When working with graphs do not attempt to put too much information on any one graph. It is better to have three or four simple graphs than one with multiple themes.

**Geometry**

- begin to work with angles and complex figures
- learn about enlargements and reductions
- work with more complex patterns, combine symmetry, movement and numbers

A variety of concrete and tactile materials can be used to begin working with angles and complex figures. Commercially available products are available to assist in this area, however, the major focus still continues to be teacher/teacher assistant prepared materials. Orientation and mobility* (refer to the Vision Resource Teacher in your area for further information) lessons are excellent functional situations to teach and reinforce angles.

**Learning About Measurements**

- learn to measure angles
- learn the formula for the area of a triangle (e.g., area = 1/2bh)
- do more complex problem solving, using measurement

Commercially prepared kits are available to assist with measurement. (Refer to the Resource list for mathematics*). Reinforcing angles in the orientation and mobility area continues to provide practice with this concept. A variety of concrete three-dimensional models can be provided to facilitate learning the objectives in this area.

**Algebra**

- learn to introduce translation between algebraic expressions and problem solving (word phrases)

Continue to build upon and learn new Nemeth symbols and formatting when dealing with algebraic expressions and formulas.
Visually Impaired Resource Guide
Mathematics Grade Seven

At this level students develop their abilities to think about relationships between numbers and ideas. They learn about ratio, proportion and percent, and expand their abilities to use charts and graphs. They learn how mathematical information can be misrepresented (bias in statistics). They begin to use algebraic equations to describe problem situations.

Students who use braille continue to be introduced to new Nemeth* symbols as they add to their repertoire of math concepts. If math skills targeted in previous levels are not attained, then time should be spent on mastery or on alternative methods. At this stage technology should be utilized for time saving activities in the classroom. The focus is on independence, accessing information, problem solving and the mastery of the Nemeth Code.

While the brailler continues to be an essential tool, competencies are also necessary in the use of electronic calculators. It is helpful to begin working with a talking scientific calculator at grade 8 as it will be required for the continued study of mathematics. Other alternatives include tactile and speech output using the Braille 'n Speak*, Braille Lite* and built-in computer calculators.

Students may continue to need concrete, functional manipulatives to understand mathematical concepts. The necessity of teacher/teacher assistant prepared materials and pre-teaching is required on an ongoing basis.

As the complexity of tasks increases, as well as the amount of technical devices used, the student who uses braille will require time to learn how to manipulate, record and analyze data.

Please refer to the Math Overview page for additional, helpful information.

Learning About Numbers

- add and subtract fractions at a more advanced level and begin to learn to multiply fractions
- begin to learn about ratio, proportion and percent
- begin to work with integers (e.g., -3 + 2 = -1)
- learn about greatest common factors
- begin to learn about exponents (e.g., 2^3 = 8)

Use available technology, when appropriate, to assist in meeting this learning outcome.
Data Analysis
- learn about sampling, bias and averages
- display information in rank order and other formats
- begin to learn about the use and misuse of statistical information

Use available technology, when appropriate, to assist in meeting this learning outcome.

Geometry
- learn about circles, triangles and angles
- learn more about similar figures and begin to learn about scale drawings
- begin to graph ordered pairs that include positive and negative numbers (e.g., place the number \(-2, 3\) on the graph)

Use available technology, when appropriate, to assist in meeting this learning outcome.

Learning About Measurements
- find the perimeter and area of composite figures
- solve problems, using ideas developed at this and previous levels
- begin to learn about surface area

Use available technology, when appropriate, to assist in meeting this learning outcome.

Algebra
- begin to evaluate and solve basic algebraic expressions (e.g., \(x + 7\), when \(x = 5\))
- begin to solve algebraic equations (e.g., \(x + 32 = 73\); what is \(x\))
- begin to use algebraic equations to describe basic problem situations

Use available technology, when appropriate, to assist in meeting the learning outcome.